Hint for Problem 9.24

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The problem is

$$T_{xx} + T_{yy} = \sin(\omega x)$$
$$T(x=0) = T(x=l) = T(y=0) = 0, T(y \to \infty) \text{ bounded.}$$

The PDE can be reduced to a homogenous one by the substitution

$$U = T + A\sin(\omega x)$$
$$U_{xx} = T_{xx} - A\omega^2 \sin(\omega x)$$
$$U_{yy} = T_{yy}.$$

Substitute into the PDE to determine the value of A for it to be homogenous:

$$U_{xx} + A\omega^2 \sin(\omega x) + U_{yy} = \sin(\omega x)$$
$$\therefore A = \omega^{-2}.$$

Now the problem becomes

$$U_{xx} + U_{yy} = 0$$
$$U(x=0) = \omega^{-2}\sin(\omega 0) = 0$$
$$U(x=l) = \omega^{-2}\sin(\omega l)$$
$$U(y=0) = \omega^{-2}\sin(\omega x)$$
$$U(y \to \infty) \text{ bounded.}$$

We can replace one of the non-homogenous boundary conditions with another substitution:

$$V = U - \omega^{-2}\sin(\omega l)\frac{x}{l} = T + \omega^{-2}(\sin(\omega x) - \sin(\omega l)\frac{x}{l})$$

so that now

$$
\begin{aligned}
&V_{xx} + V_{yy} = 0 \\
&V(x = 0) = 0 \\
&V(x = l) = 0 \\
&V(y = 0) = \omega^{-2}(\sin(\omega x) - \sin(\omega l)\frac{x}{l}) \\
&V(y \to \infty) \text{ bounded.}
\end{aligned}
$$