

ODE Initial Value Problem

Consider three gravitationally interacting particles in 2-D space, each with unit mass. We'll represent their x and y positions as the real and imaginary components of a complex number z to make the notation more compact. By Newton's law of gravitation, the equations describing their motion are

$$z_1'' = \frac{z_2 - z_1}{|z_2 - z_1|^3} + \frac{z_3 - z_1}{|z_3 - z_1|^3}$$

$$z_2'' = \frac{z_1 - z_2}{|z_1 - z_2|^3} + \frac{z_3 - z_2}{|z_3 - z_2|^3}$$

$$z_3'' = \frac{z_1 - z_3}{|z_1 - z_3|^3} + \frac{z_2 - z_3}{|z_2 - z_3|^3}.$$

The initial conditions are $z_1(0) = -z_2(0) = 0.97000436 - 0.24308753i$, $z_3(0) = 0$, $z_3'(0) = -2z_1'(0) = -2z_2'(0) = -0.93240737 - 0.86473146i$.

Solve numerically for the trajectory of the particles from $t = 0$ to $t = 7$. Plot the particle trajectories in the complex plane. (You'll find that all three particles stay on a single ∞ -shaped curve, which is rather unexpected considering that the only force acting is gravity.)